Turbulence, Heterogeneity, and Wage Earnings Inequality

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April 28, 2003

Abstract

From the mid–1960s to the mid–1980s there was an increase in wage earnings dispersion both between, as well as within educational groups in the United States. At the same time, there were increases in the amount of wage losses following displacements. This paper quantitatively investigates whether this change, interpreted as increases in human capital losses following displacements can, in an economy where different groups of workers learn new skills at different rates, lead to increases in wage earnings dispersion both between and within these groups. In the context of a model calibrated to match some selected facts of the U.S. labor market, an increase in human capital losses following a job loss can account for a considerable fraction of the increases in the wage earnings premium benefiting the faster learners, as well as in the variance of both permanent and transitory earnings of the different groups.

1 Introduction

In the last three decades there has been a well documented increase in wage earnings inequality in the United States.[1] Of particular interest for this paper is that this increase occurred not

*I thank Zvi Eckstein, Tim Kehoe, Ross Levine, James C. MacGee, and Ellen McGrattan. I also thank seminar participants at the 2002 Meeting of the Society For Economic Dynamics. This research is part of my Ph.D thesis at the University of Minnesota. I am deeply indebted to my advisor, Ed Prescott, for his constant support and helpful discussions. Financial support from Sub-Programa Ciência e Tecnologia do Segundo Quadro Comunitário de Apoio is gratefully acknowledged.

[1]For analyses of the evolution of wage earnings in the United States see for example Levy and Murnane (1992), Katz and Murphy (1992), and Gottschalk and Moffit (1994).

only across different educational groups, but also within these groups.[2] Figure 1 documents the evolution of the difference in average log weekly wage earnings of white males with and without a college degree. Figure 2 documents the evolution of the standard deviation of log weekly wage earnings of workers with and without a college degree after conditioning for years of education.[3]

A number of researchers have also claimed that *economic turbulence* has increased in the last two decades.[4] By economic turbulence I mean the average rate of skill depreciation a worker faces following a job loss. More importantly, some economists think that these changes may very well be behind the observed increase in wage earnings inequality within groups.[5]

Economic turbulence, the variable I present as a possible cause for the increase in wage earnings inequality, can also be thought of as being endogenous. In the context of this investigation it will be useful to think of it as the consequence of an increase in the depreciation rate of skills that occurs because "the way of doing things" changes more often. By this I mean that particular tasks, used in productive processes, are replaced by more efficient ones at a faster pace. This kind of exogenous technological change is admittedly difficult to document, therefore I use the proxy above. Consider why. As new technologies requiring new skills arrive more often, previously accumulated skills are less relevant, which, for workers that are displaced, is tantamount to facing a higher skill loss – higher turbulence.

This paper develops a framework to answer the following question: can an increase in economic turbulence generate quantitatively reasonable increases in wage earnings inequality, both between and within educational groups?

This research is largely motivated by the theoretical findings of Ljungqvist and Sargent (1998). Their work shows it is possible to have increases in within-group wage earnings inequality in a framework where technological shocks to the firms' production functions are absent. This contrasts with most of the existing literature.[6] Such a departure from the usual mechanisms used in the literature begs further investigation, in the present case a quantitative

[2] See Juhn, Murphy, and Pierce (1993).

[3] For the sources of the data used in the figures, please refer to section 4.

[4] See, for example, Bertola and Ichino (1995), Caselli (1999), Ljungqvist and Sargent (1998), and Violante (2002). For a survey on the increase in job displacement in particular see Kletzer (1998).

[5] Ljungqvist and Sargent (1998) show that an increase in economic turbulence can lead to increases in the dispersion of wage earnings among *ex-ante* equivalent individuals.

[6] See, for example, Acemoglu (1999), Lloyd-Ellis (1999), Galor and Moav (2000), Shi (2001), and Violante (2002).

one.

The Ljungqvist and Sargent (1998) world is one where agents are *ex-ante* homogeneous, so the authors are necessarily silent on issues regarding individuals with different abilities. Their framework – a search model with skill accumulation – is extended to include such a margin. Individuals are indexed by the rate at which they master new skills. Anybody who has sat on a school bench is aware of the fact that different people learn at different speeds. Some people are able to master new skills very quickly, while it takes others a long time to do it. This paper formalizes the very intuitive argument that in times when the rate at which skills depreciate increases, the value of the ability to learn faster also increases. It focuses on differences in the ability to learn new skills as a potentially crucial factor in understanding the changes in inequality in the last three decades.

Although this ability is observable in the model economy, this is not, in general, the case in the real world. An argument can be made, nonetheless, for the existence of a positive correlation between the ability to learn fast and education. Faster learners have a lower cost of learning, therefore, everything else being the same, will stay in school longer. This means there will be a higher proportion of faster learners among the college graduate ranks than among high school dropouts. Throughout the paper I will identify slower learners with non-college educated workers and faster learners with college educated ones.

There is an abundant literature devoted to the study of the increase in earnings inequality that started in the late 1970s in the United States. This literature can, for the most part, be divided into two broad groups: one emphasizing changes in institutions, like the decline in unionization or the decline in the minimum wage,[7] the other emphasizing technological change. In the second group, some authors have addressed the increase in inequality between educational groups,[8] others have addressed the increase in residual inequality,[9] while still others have addressed both kinds of increases.[10]

The first group treats institutional changes as exogenous. In fact, one cannot rule out the possibility that these changes and the increase in wage earnings inequality both have

[7]See, for example, DiNardo, Fortin, and Lemieux (1996) and Fortin and Lemieux (1997).

[8]Contributions in this area include, among others, Katz and Murphy (1992), Caselli (1999), Krusell, Ohanian, Ríos-Rull, and Violante (2000), and Acemoglu (2002).

[9]See, for example, Violante (2002). Within-group inequality is also referred to as residual inequality in the literature.

[10]See, for example, Acemoglu (1999), Lloyd-Ellis (1999), Galor and Moav (2000), and Shi (2001).

a common source. The drop in the unionization rate, for example, could very well be the result of increased international trade resulting from lower trade barriers, which could also be behind the increase in wage inequality according to some authors.[11] Compared to this literature, the framework presented here makes explicit the incentives facing agents, as it is derived from fundamentals.

The most popular explanation in the literature for the increase in the college premium seems to be skill-biased technological change.[12] The idea is that recent technological advances complement skilled labor, which leads to skilled workers replacing unskilled workers in some particular tasks, thus increasing wage earnings inequality between these two groups. The framework presented here is a much simpler one. Instead of relying on skill-biased technological change, the model presented here relies on another sort of technological change, an increase in the rate at which particular productive tasks become obsolete (modelled as an increase in economic turbulence.)

I find that the increase in turbulence, accounts for important fractions of the increases in the college premium as well as in the variances of both permanent and transitory earnings.[13] The magnitudes found indicate that this framework plays an important complementary role to the literature on inequality and skill-biased technological change.

The paper proceeds as follows. The next section describes the model economy. Section 3 describes the equilibrium. Section 4 describes the experiments and presents the results, and section 5 concludes.

2 The Economy

At any point in time there is measure one of individuals in the economy. The lifetime of an individual is entirely spent in the labor force. Each period, an individual has a probability α of dying, and not remaining in the labor force for next period. This means a measure α of people leave the labor force every period. To keep the population constant, a measure α of

[11] See for example Murphy and Welch (1991).

[12] This literature associates education with skills. Thus, college educated workers are often referred to as skilled workers, while high school graduates or dropouts are referred to as unskilled workers. This has to do with the fact that the former tend to be white-collar workers, while most of the blue collar workers belong to the latter group. For this reason, college premium and skill premium are sometimes taken to be synonyms.

[13] As defined in Gottschalk and Moffit (1994).

people enter the labor force every period. Moreover, the individuals' age (time since entering the labor force) is geometrically distributed, with average $(1-\alpha)/\alpha$.

At any point in time an individual has *skills* $h \in H$.

Assumption 1. *H is finite.*

Henceforth, let the minimal and maximal elements in H be denoted by h_{min} and h_{max} respectively, and let l_h denote the number of elements in H.

A *job opportunity* is a number $w \in \mathcal{W}$.

Assumption 2. *$\mathcal{W}$ has the following properties:*

(i) $\mathcal{W} \subseteq \mathbb{R}_+$*;*

(ii) $0 \in \mathcal{W}$*;*

(iii) $\mathcal{W} \setminus \{0\}$ *is compact and convex.*

Henceforth, let the minimal and maximal elements in $\mathcal{W} \setminus \{0\}$ be denoted w_{min} and w_{max} respectively.

When faced with a job opportunity, individuals can either accept it, in which case they are said to be *employed*, or reject it, in which case they are said to be *unemployed*. In the particular case when the job opportunity is $w = 0$, individuals are said to be unemployed.[14]

Unemployed agents draw job opportunities for next period, w', with a probability that depends both on their type and skill level, $p_i(h)$. The job opportunity is drawn from a distribution function that depends on their type $\Phi_i(w) = \Pr(w' \leq w)$ defined on $\mathcal{W} \setminus \{0\}$.

Let $\mathbf{B}$ denote the set of Borel subsets of $\mathcal{W} \setminus \{0\}$ and let ϕ_i be the unique probability measure, on the measurable space $(\mathcal{W} \setminus \{0\}, \mathbf{B})$, associated with Φ_i.

A job opportunity is akin to a wage rate. In this particular case, the job opportunity will work as a wage rate per skill level. The surplus generated by an individual with skill level h, matched with job opportunity w is $e = wh$.[15]

[14] In this case the accept/reject decision will turn out to be irrelevant.

[15] In Amaral (2002) I consider the more general case of $e = wf(h)$, where f is a continuous and increasing function.

There are two types of individuals, *fast learners* and *slow learners*. The subscript $i = f, s$ indexes individuals by type. Let the measure of fast learners be given by μ_f, and the measure of slow learners by μ_s, where $\mu_s + \mu_f = 1$.[16]

Individual skills evolve according to Markov chains. The transition probabilities for these Markov chains depend on whether the individual is employed or unemployed, as well as on the individual's type. It is precisely the difference in the laws of motion governing skill accumulation that distinguishes the two populations (slow learners and fast learners).

While employed, the transition probability, for an individual of type i, of going from skill level h this period to skill level h' next period is given by $\pi_i^e(h, h')$. If the individual is unemployed, this probability is given by $\pi_f^u(h, h')$. The job match may cease to exist, which happens with probability λ_i. In this case, the transition probability is given by $\pi_f^t(h, h')$.

The evolution of the individual state variables is as follows. At the beginning of the period, an individual has state (i, h, w). If $w = 0$, the individual does not enter a job match and obtains earnings equal to zero. Next period's skill level is h' with probability $\pi_i^u(h, h')$, while next period's job opportunity is $w' \in W' \subseteq \mathcal{W} \setminus \{0\}$ with probability $p_i(h)\phi_i(W')$, or $w' = 0$ with probability $(1 - p_i(h))$. If $w > 0$, the individual decides whether to accept or reject this job opportunity. In the case where the individual rejects w, she does not enter a job match and obtains earnings equal to zero. Next period's skill level is h' with probability $\pi_i^u(h, h')$, while next period's job opportunity is $w' \in W' \subseteq \mathcal{W} \setminus \{0\}$ with probability $p_i(h)\phi_i(W')$, or $w' = 0$ with probability $(1 - p_i(h))$. In the case where the individual accepts w, she enters a job match and obtains earnings equal to wh. If this match dies, which occurs with probability λ_i, next period's skill level is h' with probability $\pi_i^t(h, h')$, while next period's job opportunity is $w' = 0$. If this match survives, which occurs with probability $(1 - \lambda_i)$, next period's skill level is h' with probability $\pi_i^e(h, h')$, while next period's job opportunity is $w' = w$.

The inclusion of the possibility of a match death is the model's counterpart of a job loss. By assumption, after a match dies, the individual is unemployed for at least one period. This assumption was made for convenience and it does not influence the results. The match death probability and subsequent skill evolution are crucial in modelling the changes in turbulence.

In what follows, I state some assumptions about the individual's transition probabilities.

Assumption 3. *Laws of motion for employed:*

[16] These measures are time invariant. Individuals do not choose to be fast or slow learners. This assumption is an important one and is discussed in the conclusion.

(i) $\pi_i^e(h,h') = 0$ *for all* $h' \neq h, h+1$*, and* $i = f, s$*, where* $h+1$ *is the smallest element in* H *strictly greater than* h*;*

(ii) $\pi_f^e(h,h) \leq \pi_s^e(h,h)$ *for all* h*.*

Assumption 3.*(i)* states that an employed individual's skill level next period will either be the same as in the current period or the skill level immediately above. For employed individuals, skill level evolution is weakly increasing.[17] This implies that once an individual reaches the maximum skill level, h_{max}, she will remain there while employed. This assumption, which is made for simplicity, is the reason why a fixed fraction of individuals, α, are assumed to leave the labor force every period, while the same fraction enters the labor force with the minimum skill level. Finally, assumption 3.*(iii)* states that, while employed, fast learners have a higher probability of jumping to the next skill level than slow learners, hence the names for each of the types.

Assumption 4. *Laws of motion for unemployed:*

(i) $\pi_i^u(h,h') = 0$ *for all* $h' > h$*;*

Assumption 4.*(i)* states that an unemployed individual's skill level next period will be smaller than or equal to the current period's skill level. For unemployed individuals, skill level evolution is weakly decreasing. This implies that once an individual reaches the minimum skill level, h_{min}, she will remain there while unemployed.

Assumption 5. *Laws of motion for job losers:*

(i) $\pi^t(h,h') = 0$ *for all* $h' > h$*.*

Assumption 5.*(i)* states that a job loser's skill level next period will be smaller than or equal to the current period's skill level. This reflects the extent to which accumulated skills are still relevant following job losses. It also reflects the extent to which "the way things are done" changes. One of the environment changes that will be considered is to make the expected skill level following a match death smaller.

[17] Topel (1991) finds evidence in support of the view that the accumulation of specific capital is an important ingredient in determining life-cycle earnings.

Individuals' period utility is linear in consumption and they maximize future expected discounted utility. Their objective is:

$$E_t \sum_{j=0}^{\infty} \beta^j (1-\alpha)^j c_{t+j}, \tag{1}$$

where c is consumption and β is the discount factor.

The linear utility assumption allows me to ignore the private bonds market. Without loss of generality I can shut down this market.

The individuals' earnings are assumed to be equal to the surplus generated by the match. This is equivalent to have the individual own the job match. The purpose of this assumption is to avoid issues regarding surplus sharing that are fundamentally linked to the nature of the firm and are not the main focus of my analysis.[18]

3 Equilibrium

Let $V_i^u(h)$ denote the discounted expected utility of an agent of type i, with skill level h, that has a job opportunity $w = 0$, akin to unemployment. $V_i^u(h)$ is given by:

$$V_i^u(h) = \beta(1-\alpha)\left\{\sum_{h'} \pi_i^u(h,h')\left[p_i(h')\int V_i(h',x)\,d\Phi_i(x) + (1-p_i(h'))V_i^u(h')\right]\right\}, \tag{2}$$

for $i = f, s$. $V_i(h,w)$ denotes the discounted expected utility of an individual of type i, with skills h, that has a job opportunity $w > 0$, and is given by:

[18] This does not mean the results would be the same otherwise. Generally, the literature assumes the surplus generated by the match is shared according to a generalized Nash bargaining rule. See for example Pissarides (2000), section 1.4. In this case my assumption is tantamount to having the worker's Nash bargaining weight equal 1.

$$
\begin{aligned}
V_i(h,w) \;=\; & \max\Big\{ V_i^u(h), \quad wh + \\
& \beta(1-\alpha)\Big[(1-\lambda_i)\sum_{h'} \pi_i^e(h,h')V_i(h',w) + \\
& \lambda_i \sum_{h'} \pi_i^t(h,h')V_i^u(h')\Big]\Big\}, \quad \text{for } i = f, s.
\end{aligned} \tag{3}
$$

Equation 3 states that an individual of type i, with skill level h, who receives a job opportunity $w > 0$, can accept it or reject it. In case of rejection, the individual is unemployed this period, and the value of that is given by $V_i^u(h)$. In case of acceptance, earnings this period are wh. Next period, with probability $(1-\lambda_i)$, the same job opportunity is offered, so the expected value is taken only over all possible skill levels next period; with probability λ_i the match dies, so the individual is unemployed next period, which has a value of $V_i^u(h')$ for each possible skill level next period, h'.

The following proposition states that the optimal policy associated with equation 3 exists and is unique. All the proofs are in Amaral (2002).[19]

Proposition 6. *Under assumptions 1 and 2, the solution to equation 3 exists and is unique except for a set of measure zero.*

To address characterization, the next proposition states that the optimal policy associated with equation 3 is of the reservation wage type. If the job opportunity an individual of type i and skill level h faces is greater than or equal to $\underline{w}_i(h)$, then it is optimal to accept it, otherwise it is optimal to reject it.

Proposition 7. *Under assumptions 1 and 2, the optimal policy associated with equation 3 is of the reservation wage form. For any $h \in H$ there exist numbers $\underline{w}_i(h)$, $i = f, s$, such that an agent of type i with skills h, will accept job opportunity w if $w \geq \underline{w}_i(h)$, and reject it if $w < \underline{w}_i(h)$. Furthermore, the solution to (3) is nondecreasing in w.*

The following proposition further characterizes the solution to (3). It states that the solution is a continuous piecewise linear function of the job opportunity w.

[19]The proofs are included in the appendix for the referees' convenience.

Proposition 8. *Under assumptions 1, 2, and 3, $V_i(h, \cdot)$ is a continuous piecewise linear function of w.*

Given the individuals' optimal policies, $\underline{w}_i(h)$, the next step is to make explicit the laws of motion governing the transitions between different states. Let $\mu_t(i, h, W)$, where $W \subseteq \mathcal{W}\backslash\{0\}$, denote the period t measure of individuals of type i, with skill level h, and a strictly positive job opportunity in $w \in W$, while $\mu_t(i, h, 0)$ denotes those individuals with job opportunity $w = 0$. For every period t,

$$\sum_i \sum_h \left[\mu_t(i, h, 0) + \int_{w_{min}}^{w_{max}} \mu_t(i, h, x)\, dx \right] = 1. \tag{4}$$

For this equation to hold over time, the measure of individuals entering the labor force has to equal the measure leaving it, given by α. I assume that those people entering the labor force do so with a skill level of $h = h_{min}$ and a job opportunity $w = 0$.

Letting $\mu_{t+1}(i, h', W')$ denote next period's measure, the evolution of these measures is given by:

$$\begin{aligned} \mu_{t+1}(i, h', W') &= (1-\alpha)\Bigg\{ (1-\lambda_i) \sum_h \pi_i^e(h, h') \mu_t(i, h, W') \chi_{(w' \geq \underline{w}_i(h))} \\ &+ \phi_i(W') p_i(h') \sum_h \pi_i^u(h, h') \mu_t(i, h, 0) \\ &+ \phi_i(W') p_i(h') \sum_h \pi_i^u(h, h') \int_{w_{min}}^{\underline{w}_i(h)} \mu_t(i, h, x)\, dx \Bigg\} \end{aligned} \tag{5}$$

The three lines in equation 5 highlight the fact that individuals that are part of the measure $\mu_{t+1}(i, h', W')$ have three possible origins regarding their previous period's state. The first line refers to those individuals that were employed the previous period at job opportunity $w' \in W'$, and evolved to skill level h'.[20] Only a fraction $(1 - \lambda_i)$ of these actually gets the same job opportunity. The second line refers to those individuals that had job opportunity $w = 0$ the previous period and evolved to skill level h'. Only a fraction $\phi_i(W')p_i(h')$ will have

[20] $\chi_{(w' \geq \underline{w}_i(h))}$ is an indicator function that equals one when the argument is true, otherwise it is zero. This captures the employed only.

an offer of $w' \in W'$. Finally, the third line captures all those that had a strictly positive job opportunity the previous period, but rejected it and evolved to skill level h'. Again, only a fraction $\phi_i(W')p_i(h')$ will have an offer of $w' \in W'$.

Some individuals loose their job, while others simply do not get a job offer after rejecting one or after having lost their job. These are the people that have $w = 0$. The evolution of the measure of these individuals is given by:

$$\begin{aligned}
\mu_{t+1}(i,h',0) &= \alpha\chi_{(h'=h_{min})} + (1-\alpha)\Bigg\{\lambda_i \sum_h \pi_i^t(h,h') \int_{\underline{w}_i(h)}^{w_{max}} \mu_t(i,h,x)\,dx \\
&+ (1-p_i(h'))\sum_h \pi_i^u(h,h')\mu_t(i,h,0) \\
&+ (1-p_i(h'))\sum_h \pi_i^u(h,h') \int_{w_{min}}^{\underline{w}_i(h)} \mu_t(i,h,x)\,dx\Bigg\} \qquad (6)
\end{aligned}$$

The first term on the right-hand-side of equation 6, $\alpha\chi_{(h'=h_{min})}$, is the measure of people entering the labor force for the first time. The first line inside the curly brackets includes all individuals that were employed in the previous period, but lost their job and evolved to skill level h'. The second line refers to those individuals that had job opportunity $w = 0$ the previous period and evolved to skill level h'. Finally, the summation in the third line captures all those that had a job opportunity $w > 0$ the previous period but rejected it and evolved to skill level h'.

I will consider only *steady-state equilibria*. I do this because both the college differential and the residual wage dispersion seem to be relatively stable (although at different levels) in the late 1960s and in the late 1980s to early 1990s. In contrast, in between the two periods, these variables show a positive trend, suggesting that the former steady-state was perturbed by some kind of shock followed by dynamics leading to the latter steady-state.[21]

The following is the definition of steady-state equilibrium.

Definition 9. *A* ***steady-state equilibrium*** *is a set of reservation job opportunities, $\underline{w}_i(h)$, and associated invariant probability measures, $\mu(i,h,W)$, $W \subseteq \mathcal{W} \setminus \{0\}$, and $\mu(i,h,0)$, such*

[21]See figures 1 and 2. Beaudry and Green (2002) also support this view.

that:

1. $\underline{w}_i(h)$ *are the optimal policies for (3), for each type* i*, and each skill level* h*;*
2. *given* $\underline{w}_i(h)$*,* $\mu(i, h, W)$*, and* $\mu(i, h, 0)$ *solve (4), (5) and (6).*

Note that the measures $\mu(i, h, W)$ and $\mu(i, h, 0)$ are the invariant measures associated with equations 5 and 6.

The next proposition shows existence and uniqueness of the above equilibrium.

Proposition 10. *Under assumptions 1, 2, 3, 4, and 5, the equilibrium in definition 9 exists and is unique.*

The framework developed will now be used to conduct experiments that will help determine whether increases in the average rate of skill depreciation following job losses can help us understand the observed increase in wage earnings dispersion both between, and within the two groups.

4 Experiments

In this section I conduct experiments that can be thought of as resulting from an increase in the rate at which tasks become obsolete. The average rate of skill depreciation following a job loss increases and the economy moves from a steady-state with relatively low turbulence to a steady-state with relatively high turbulence.

I use data for two purposes. To establish a benchmark against which to compare the results of the experiments involving the model economy and to calibrate the model economy. The latter is postponed to the next subsection and here I will describe the data and measures used.

I use the Current Population Survey (CPS) March files, as well as two supplements to the CPS: the Displaced Workers Survey (DWS) and the Job Tenure Supplements (JTS). I consider full-time, full-year, white male workers between 20 and 64 years of age, that are not self employed. I consider this particular set of workers because in the model economy there is no choice regarding how much time to work. This is intended to avoid biases caused by workers that are heterogeneous in dimensions that the model economy ignores. This is also the reason

I look at white males only: the model economy is unable to capture wage dispersion stemming from gender and racial differences, and this is the most numerous gender/race subgroup.

The measure of earnings adopted is the log of weekly wages. The weekly wage earnings are deflated by the 1982-84 Consumer Price Index for urban consumers published by the Bureau of Labor Statistics. This is the most common measure of wage earnings in the literature.

I divide this sample into two distinct populations, those with a college degree and those without a college degree.[22] Relative to the model economy, I identify college graduates with fast learners, and the rest of the population with slow learners. Besides the arguments presented in the introduction regarding this choice, figure 3 presents hourly wage earnings by tenure for these two groups computed from the JTS.[23] This figure supports the view that, while on the job, college workers seem to learn faster, to the extent that a higher growth in wage earnings reflects increases in the skill level. Note that while it takes non-college educated workers 28 years, on average, to attain the maximum wage earnings, it takes college educated workers only 23 years.[24] This will be used later to calibrate the laws of motion governing the employed workers' skill level, $\pi_i^e(h, h')$.

From the data, I determine benchmarks for changes in: i) a measure of relative wage earnings and; ii) a measure of within-group wage earnings dispersion. The measure of relative wage earnings I use is the college differential, which is defined as the difference in average log weekly wages between college educated individuals and those without a college education. This measure approximates the college premium (defined as the ratio of average weekly wages minus one), for small enough premia. This is presented in Figure 1. Note that the college differential was much lower in the late 1960s than in the late 1980s and early 1990s. Furthermore, during these two periods, this measure remained relatively stable.

The measure of wage earnings dispersion within groups I use is the standard deviation of log weekly wage earnings for each of the two groups. I regress the log weekly wage earnings on years of education, and then calculate the standard deviation of the residuals of this regression. These are shown in Figure 2. Note that the residual wage dispersion was much lower in the late 1960s than in the late 1980s and early 1990s. Furthermore, during these two periods, this measure remained relatively stable.

[22] By a college degree I mean having completed at least a BA, AB, or BS.

[23] Details regarding the data for the figures and tables presented are in the appendix.

[24] Figure 3 does not control for potentially important factors such as firm size or industry, so it should be interpreted with reservations. Nonetheless, Murphy and Welch (1990) obtain similar results.

Another important dimension in which residual dispersion increased is reported by Gottschalk and Moffit (1994). They find that between 1970-78 and 1977-87, both the variance of permanent earnings, as well as the variance of transitory earnings increased. Focusing on the white males in the Michigan Panel Study on Income Dynamics (PSID)[25], they decompose individual log yearly wage earnings over time into a permanent and transitory component: $y_{it} = \mu_i + \nu_{it}$. They proceed to compute the average (across individuals) variance of the permanent component, μ_i, as well as the average (across individuals and over time) variance of the temporary component, ν_{it}, over the two periods.

Table 1 presents the changes in summary statistics against which the changes in the model's counterparts to these statistics will be compared.

The DWS data is used to compute wage earnings losses following displacement, as well as the fraction of workers that are displaced. There are two important qualifications that need to be made clear. One is that not all the workers that loose their job are displaced, some are fired, and some others quit.[26] In the next subsection I make clear what kind of assumptions are made regarding workers that are fired or quit. The second point to note is that the DWS only started in 1984, so it is impossible to know, using this survey, how the wage earnings losses and the job loss rate evolved from the late 1960s to the late 1980s and early 1990s. In the next subsection I examine evidence on this issue from other sources.

A potentially important determinant in the law of motion governing the skill level following a job loss is whether the agent changes occupations (is a switcher) subsequently or not, the idea being that job losers that switch should face higher wage earnings losses, since they loose both occupation-specific capital (associated with the occupation change), as well as person-specific capital (associated with an unemployment spell). To be able to accommodate for this possibility, let γ_i denote the probability that an individual of type i changes occupations, and let $\pi^{ts}(h, h')$ and $\pi^{tns}(h, h')$ denote the probability of going from skill level h to h' following a job loss, for switchers and non switchers, respectively. Then, equation 3 becomes:

[25] Gottschalk and Moffit (1992) argue that the PSID tracks the CPS quite well in terms of the trends in the variance of log wage earnings.

[26] A worker is considered displaced, according to the 1984-92 DWS, if she was involuntarily separated from her job due to a plant closing, an employer going out of business, a layoff from which [the worker] was not recalled, a seasonal job ended, or failure of a self-employed business. The 1994-98 DWS, takes into account only the first three reasons.

$$
\begin{aligned}
V_i(h,w) \quad = \quad & \max\Bigg\{V_i^u(h), \quad wh + \\
& \beta(1-\alpha)\Bigg\{(1-\lambda_i)\sum_{h'}\pi_i^e(h,h')V_i(h',w) + \\
& \lambda_i\Big[\gamma_i\sum_{h'}\pi_i^{ts}(h,h')V_i^u(h') + (1-\gamma_i)\sum_{h'}\pi_i^{tns}(h,h')V_i^u(h')\Big]\Bigg\}\Bigg\}, \\
& \text{for } i = f, s.
\end{aligned} \tag{7}
$$

Note that the introduction of switchers and non-switchers does not change any of the results in section 3.

Table 2 presents three-year displacement rates for switchers and non-switchers.[27] The displacement rates faced by non-college educated workers are considerably higher than those faced by college educated ones. This suggests that non-college educated workers are the first out the door when firms have to cut jobs. This may be because the costs of laying off college educated workers are higher, which is certainly true if one thinks about severance payments, but most importantly for this line of research is that firms seem to recognize that college educated workers can adapt more easily to new technologies. Another fact that stands out is that the displacement-non-switching rates are considerably smaller than their switching counterpart. This tells us that among those workers that are displaced, most of them (63.8% of the college educated and 67.7% of the others) change occupations. Such a statement should not be confused with a statement saying that most of the workers that separate from a job change occupations. Recall that I am just looking at displaced workers, thus ignoring quits and firings. These facts will be used in the next section to calibrate the probability of a job loss, λ_i, and the probability of subsequently switching , γ_i.

Regarding the change in wage earnings workers face after being displaced and switching occupations (or not), the statistic I compute is the difference in log weekly wage earnings between the current and the previous job. In the context of the model, this is an indicator

[27] I present three-year displacement rates so that they are directly comparable to Farber (1997). Details regarding the computation of these rates are in the appendix. In Amaral (2002) I present more details about the data, namely a demographic breakdown.

of the skill change workers face when they are displaced. This is used in the next section to calibrate the laws of motion for skills after displacement: $\pi_i^{ts}(h,h')$ and $\pi_i^{tns}(h,h')$.

Table 3 presents the data on the log wage earning difference following displacement. As expected, the wage earnings losses suffered by displaced switchers are, on average, higher than those suffered by displaced workers that subsequently find a job in the same occupation. Also, note that the wage earnings losses faced by the non-college educated workers are, on average, higher than those faced by the college educated ones. This is more so for switchers than for non-switchers. A pattern that holds on average is that among displaced workers, the non-college switchers are the ones that loose the most, followed by the college switchers, followed by non-college non-switchers, finally followed by college non-switchers.

4.1 Increases in Turbulence

As discussed in the introduction, a number of authors have claimed that the average rate of skill depreciation following a displacement, termed here economic turbulence, has increased in the last two decades.[28] The most solid evidence for this is provided in Violante (2002). Using the PSID, he measures white males' wage losses upon displacement (computed as the difference between the hourly wage on the new job N years after separation and the last wage earned before separation). He divides the sample into two periods: 1970-80 and 1981-90 and concludes that the wage losses one year after displacement in the second period are higher than those in the first period by roughly 10 percent. There are two problems in mapping this finding into the model presented here. The first is the slight difference in time frames. For the purpose of this model, the first period Violante looks at does not coincide with the first period I look at, which is 1963-69. Can we make any inference about the bias caused by this difference? If anything, given the qualitative evidence raised most notably by Caselli (1999), the increase in the wage losses upon displacement would be even higher.

From Low Turbulence to High Turbulence

In this first experiment, I calibrate the model using the data reported above and assume, following Violante (2002), that wage losses upon a job loss grew by 10 percent from the 1960s to 1981-97, both for switchers and non-switchers.

[28] Ljungqvist and Sargent (1998) were, to my knowledge, the first to call turbulence to the average rate of skill depreciation following a job loss.

In terms of calibration, I set the model period to one week. Both firms and workers make very frequent decisions about hiring and looking for jobs. One week seems like a good approximation. Also, most of the literature reports either hourly wages or weekly wages, having a period be a week makes the results directly comparable.

In the model economy, a worker's life coincides with her worklife, so I set the parameter α such that the expected worklife of a worker is 41 years. This number is reported by Millimet, Nieswiadomy, Ryu, and Slottje (2002) for both college educated, as well as high school educated white males in the United Sates. I set β, the discount factor, such that the annual interest rate is 4 percent.

I assume that $\mathcal{W}\backslash\{0\}$ contains 100 points, evenly distributed between 10 and 1000. The set H is assumed to contain 21 points, evenly distributed between 1 and 2. This means that the range for the strictly positive weekly wage earnings is between 10 and 2000. I experimented with increasing the size of this set, as the observed wage range is wider, but this did not change the results.

With respect to the law of motion for the skill level when the worker is employed and does not loose the job, $\Pi_i^e = [\pi_i^e(h, h')]$, regardless of the skill level, there is a constant probability of either staying at h, $\pi_i^e(h, h)$, or transiting to $h + 1$, $1 - \pi_i^e(h, h)$. I select this probability such that, conditional on continuous employment, the fast workers achieve the maximum skill level in 23 years, while slow workers do it in 28 years. These values are taken from figure 3.

With respect to the law of motion for the skill level when the worker is unemployed, $\Pi_i^u = [\pi_i^u(h, h')]$, regardless of the skill level, there is a constant probability of either staying at h, $\pi_i^u(h, h)$, or transiting to $h - 1$, $1 - \pi_i^u(h, h)$. I let $\pi_i^u(h, h) = 0.9$, this means that, conditional on a year of continuous unemployment, both types of workers loose between 0 and 18.3 percent (depending on their skill level). Keane and Wolpin (1997) estimate that workers loose between 10 percent (blue collar) and 30 percent (white collar) of their skills following a one year spell of unemployment. One could argue that most college graduates are white collar workers, while most blue collar workers are non-college educated. I opted for giving both groups the same value since the results are not sensitive to this number.

The job loss rate must include workers that are fired in addition to those that are displaced. Recall that I only have these rates available for displaced workers. Using data from the Employment Opportunity Pilot Project for 1980, Campbell III (1997), concludes that displacement rates and firing rates are very similar. For non-college educated male workers the probability of being fired is 8.8 percent higher than the probability of being displaced. For college educated workers, the probability of being fired is 6.2 percent higher than the probability of being displaced. Contrarily, using PSID data from 1976 to 1991, Valletta (1998) concludes that for males the probability of being displaced is 4.4 percent higher than the probability of being fired. Given the magnitudes at hand (a small percentage of a probability around 10 percent), and in the interest of simplicity, I will assume that the probability of firing is the same as that of displacement. From table 2, the total probability of displacement for non-college workers is 9.9 percent (6.7 percent are switchers and 3.2 percent are non-switchers). This means the total three year probability of a job loss for non-college workers, λ_s, is 19.8 percent. Of these, 67.7 percent are switchers (γ_s) and the rest are non-switchers. The total three year probability of displacement for college workers is 5.8 percent percent (3.7 percent are switchers and 2.1 percent are non-switchers). This means the total probability of a job loss for college workers, λ_f, is 11.6 percent. Of these, 63.8 percent are switchers (γ_f) and the rest are non-switchers.

Abusing language, I will say that each of the remaining parameters are calibrated to a particular statistic. In fact, they affect the statistics they are calibrated to jointly. Below, I paired parameters with the statistics they influence more strongly.

As discussed before, and following Violante (2002), I assume that the wage earnings losses following a job loss grew by 10 percent from the 1960s to 1981-97. Violante's estimation considers switchers and non-switchers together. I assume that the change was the same for both types.

The calibration of the law of motion for the skill level following a displacement, $\Pi_i^t = [\pi_i^t(h,h')]$, follows Ljungqvist and Sargent (1998).[29] Recall from assumption 5, that, for $h' > h$, $\pi_i^t(h,h') = 0$. For $h' \leq h$, let $\pi_i^t(h,h')$ be proportional to the left side of a normal distribution that has mean h and standard deviation σ_i^t. Formally, for $h' \leq h$, $\pi_i^t(h,h') \propto f_{N(h,\sigma_i^t)}(h')$. This means that for $h' \leq h$, $\pi_i^t(h,h')$ is a discrete approximation to the left side of a normal that has been truncated and resized to integrate to one. Under this modelling approach, the parameter σ_i^t is an indicator for economic turbulence.

When a worker looses her job, a higher σ_i^t implies a lower expected skill level next period. I then set σ_i^t such that the average log wage earnings loss following a job loss, for each group,

[29]For notational simplicity I will omit the superscripts relative to switchers and non-switchers, meaning that where it reads π_i^t, it should read π_i^{ts} and π_i^{tns}.

in the model economy, is the same as the average log wage earnings loss faced by college and non-college educated white males following a job loss and switching (or non-switching) occupations. This formulation assumes that people that are displaced and people that are fired suffer the same skill losses. This is a shortcut since I do not have data for wage earnings losses following firings (rather than displacements). Recall from table 3 that the average wage loss for white male switchers in the latter period was 7.1 percent for the college educated workers and 11.9 percent for the non-college educated. The wage losses for the two groups of switchers in the 1960s are then set to 6.5 percent and 10.8 percent, respectively. For the non-switchers, the wage losses in 1981-97 averaged 3 percent for the college educated and 4.5 percent for the non-college educated. Given the above assumption, the wage losses for the non-switchers in the 1960s are set to 2.7 percent for the college educated and 4.1 for the non-college educated.

Regarding the probability of receiving a job opportunity, $p_i(h)$, I assume that it is a strictly increasing linear function of h and that $p_i(h_{max}) = 1$. This way, the only parameter that needs to be calibrated is $p_i(h_{min})$. I set this parameter such that the average wage earnings of the workers with the smallest skill level in the model economy, in each group, are the same as the average wage earnings of white males with the smallest skill level in the U.S economy. I do not know the workers' skill level in the U.S. economy, but I know that in the model economy it takes

$$\frac{1-\pi_f^e(h_{min}, h_{min}+1)}{\pi_f^e(h_{min}, h_{min}+1)} = \frac{0.98355}{0.01645} = 59.79$$

weeks for an employed college educated worker to reach the second skill level, while this number is

$$\frac{1-\pi_s^e(h_{min}, h_{min}+1)}{\pi_s^e(h_{min}, h_{min}+1)} = \frac{0.98645}{0.01355} = 72.8$$

weeks for slow workers. Given the average log weekly wage earnings by experience level, I compute an upper bound (since I am assuming continuous employment) on the average log weekly wage earnings that workers with such experience make. These numbers are, in 1982-84 log US$, 5.46 for the non-college educated workers and 5.81 for the college educated ones in the 1963 to 1969 period. I then calibrate $p_f(h_{min})$ and $p_s(h_{min})$ such that, in the model economy, the average log weekly wage earnings of the fast and the slow, with the lowest skill level, equal these numbers, respectively.

I construct the distribution from which the workers draw wages, ϕ_i, from a normal distribution with mean μ_i^w and standard deviation σ_i^w. Since the support of this distribution is the finite set $\{10, 20, ..., 1000\}$, I make the distributions discrete and rescale them so that they integrate to one. I set the means of these distributions, μ_i^w, such that the average log weekly wage earnings in the model economy, for fast and slow workers, equal the average log weekly wage earnings of U.S. white male workers that have a college education (6.33 log 1982-84 US$) and those that do not (5.94 log 1982-84 US$) over the 1963-69 period. I follow Ljungqvist and Sargent (1998) and set the standard deviation of this distribution, for both types, to 100.[30] The complete calibration is presented in table 4.

The results of this experiment are reported in table 5. The experiment generates two types of statistics. One type concerns the steady-state distribution of log wage earnings (a cross-section). These statistics are contained in the first three lines of table 5 and are respectively, the college differential and the cross-section standard deviation of log weekly wage earnings for the two groups. The second type of statistics is computed from an artificial panel and is contained in the last four lines of table 5. These are the variance of the permanent component of earnings for both types, $var(\mu_j)_i$, and the variance of the transitory component of earnings for both types, $var(\nu_j)_i$. They were computed from a panel of 20,000 individuals (10,000 of each type) simulated over 10 years. Since Gottschalk and Moffit (1994) purge their statistics of life-cycle effects, I consider only individuals with over 40 years of experience, which should be enough to remove any life-cycle effects since, on average, it takes 28 (23) years for fast (slow) individuals to reach the maximum skill level.

The model generates an increase in the college differential that is of a similar order of magnitude as that in the data. The increase in the college differential in the model is roughly 11 percent of the one in the data. This confirms the intuition that in times when skills depreciate faster, being able to learn relatively faster commands a higher premium than in times when skills depreciate slower. This magnitude is comparable to the ones obtained in the literature.[31]

The next thing to note is that the effects of a plausible change in turbulence on within-group wage earnings dispersion in the cross-section are negligible. In fact, for the slow, the wage earnings dispersion decreases slightly. There are at least two easy, although, in this context, non-informative, ways of having the wage dispersion within the two groups increase. One is to subdivide the groups further. For example, within the slow, one could have high

[30] In Amaral (2002) I show that the results are not sensitive to this choice.
[31] See, for example, DiNardo, Fortin, and Lemieux (1996).

school dropouts, high school graduates, and college dropouts learn at different rates. This would increase inequality within this group of three kinds of agents because of changes in average wages between the three subgroups, not because of changes within each of the three subgroups. Another possible way one can reconcile this result with the data is to have heterogeneity in learning ability within the two groups. In this context the two groups would both contain fast and slow learners, but the group termed ”fast” would contain a higher fraction of fast learners.

If one looks at the panel statistics (in the last for rows of the table) though, and compares them to the Gottschalk and Moffit (1994) numbers, the results are very encouraging. The change in turbulence is able to account for 50 (12) percent of the change in the variance of the permanent component of earnings for the (non) college educated individuals. It is also able to account for 18.6 (2.5) percent of the change in the variance of the permanent component of earnings for the (non) college educated individuals. The intuition for these increases is that when faced with a job loss, individuals face more uncertainty regarding their subsequent skill level, and fall, on average, to a lower skill level than when turbulence is smaller. The net effect is not obvious, since there is a force pulling in the opposite direction: the fact that jobs are less valuable in the environment with higher turbulence, also means that the reservation wages are higher, therefore the interval of accepted wages is narrower.

Note that these results are compatible with the lack of change in the cross-sectional dispersion, since for the panel statistics, the sample is restricted to experienced workers, so as to abstract from life-cycle effects. The cross-sectional results together with the panel results, seem to indicate that for less experienced workers, the life-cycle effects counteract the effects of an increase in turbulence resulting in stagnant dispersion.

What is innovative about this framework is precisely the fact that it allows one to address both types of increases in dispersion, within and between groups, with success on both margins.

From No Turbulence to High Turbulence

In this experiment it is assumed that there was no economic turbulence in the 1960s, meaning that a worker’s skill level following a job loss remained the same. Turbulence in the 1980s and 1990s is set at the level observed in the data. This is done with the purpose of understanding how much of the changes in the data, in the limit, can this framework account for, which is important if one thinks the estimate for the increase in turbulence obtained from Violante (2002), 10 percent, is a conservative one.

The calibration strategy is the same as the one reported for the previous experiment and is summarized in table 6.[32]

The results for this experiment are summarized in table 7. Changes in turbulence can account for up to eighty percent of the increase in the college differential, which emphasizes the potential of this framework. In terms of the cross-sectional residual dispersion, one concludes that changes in turbulence cannot, in this framework, account for the increases in the data. Below I try to understand why. In contrast, changes in turbulence have the potential to account for all the increases in the dispersion of the permanent component of wage earnings. This potential seems to be more limited regarding the increases in the dispersion of the temporary component of wage earnings.[33]

The exaggerated change in turbulence in this experiment is ideal to understand what the forces at work when turbulence increases are. In the experiments, moving from a steady-state with low turbulence to a steady-state with high turbulence always decreased average log wage earnings, regardless of the learning speed. Why is this so? On one hand, in general, an increase in turbulence makes employment become relatively less attractive than unemployment (future job opportunities) in such a way that the optimal reservation wage schedule is uniformly (weakly) higher.[34] On the other hand, the fact that the reservation wage schedule is higher, means less people are employed, which in turn means a lower average skill level. Reinforcing this effect is the fact that when workers loose their job, they slide further down the skill range in times of higher turbulence. The last two effects seem to dominate, leading to lower average wage earnings when turbulence increases. The increase in the college differential happens because in times when skills depreciate faster (in a stochastic sense), the ability to learn faster earns a higher premium. This means that although the reservation wage schedule of the slow increases more than that of the fast, this effect is dominated by the fact that the

[32] I only report the parameters that changed from the previous experiment.

[33] Note, in particular, that the changes in the dispersion of the temporary component of wage earnings are not monotonic in the changes in turbulence.

[34] The statement has the qualification "in general", meaning for most parameter values. This is because the value of unemployment is also dependant on the value of employment, so anything that lowers the latter, lowers the former. For some parameter values it happens that, for a given skill level, the reservation wage under higher turbulence is smaller than the reservation wage under lower turbulence. The tradeoff is the usual substitution versus income effect, most of the times the former dominates.

average skill level of the slow falls by more than that of the fast.

The intuition for the lack of change in the cross-sectional residual wage earnings dispersion is slightly more contrived. Although the higher rate of skill depreciation pushes the earnings distribution towards smaller values, "squashing" it against the lower bound of the support, the overall effect on dispersion depends on the initial earnings distribution. In fact, if the initial distribution is relatively concentrated on high earnings, dispersion increases with increases in turbulence, but as the initial steady-state average skill level decreases to lower, more realistic, values this ceases to be true. In the next section it is shown how this depends crucially on the calibration of the law of motion of skills while employed. This suggests that this framework might not be the more appropriate to think about the cross-sectional residual wage earnings dispersion, as such a simple model is unable to reproduce the actual earnings distribution in the 1960s.[35]

Ljungqvist and Sargent's (1998) Experiment

The Ljungqvist and Sargent (1998) experiment is replicated here with the purpose of understanding why this framework cannot replicate the increase in cross-sectional residual dispersion. In their model there is only one group of workers and there is no distinction between switchers and non-switchers. That aside, the only other difference is that in their world, workers decide on (costly) search intensity, which affects the probability of obtaining a job opportunity, while in mine, they are faced with an exogenous probability of getting an offer. As it will become apparent, this does not influence the results.[36]

In table 8 I present the equivalent of their (1998) calibration for a one week period (they considered a two week period). To this, I add a probability of getting a job offer, given the minimum skill level, of $p(h_{min}) = 0.4$.[37] I also report the facts they calibrated to. The question marks (??) mean that the authors assigned values to parameters without mentioning what they were matching.

Table 9 presents the results the increase in turbulence they consider. The important thing to look at is the increase of 5 percent in the standard deviation of the log wage earnings,

[35] Note that the calibration replicates only a pair of moments.

[36] Ljungqvist and Sargent (1998) consider two types of economies, one with unemployment compensation, another without it, I am obviously making comparisons to the latter. The purpose of including varying search intensity in their work was for it to interact with the unemployment subsidy.

[37] This was not calibrated, it was the number that better matched their results.

which contrasts with my results, and is in the same order of magnitude as the increases in the data, reported in table 1. These increases where of 8.8 percent for the college educated workers and 22.3 percent for the non-college educated ones. I argue that the results regarding this increase in within group wage earnings dispersion depend crucially on the calibration, so a discussion of the parameter values is in order. I have nothing to say about the probability of survival α, the discount factor β, or the probabilities governing the skill's law of motion while a worker is unemployed, $\pi^u(h, h')$. The values for the mean and standard deviation of the distribution of wage draws, μ^w, and σ^w, lead to higher than observed average wages, but they do not influence the result.

The value of λ seems excessively high. The authors report that this is calibrated to reflect the observation that, on average, a worker holds 10 jobs during her lifetime. The problem with using this observation is that roughly half of the job separations occur because of voluntary quits, which one would think are not associated with skill losses as large as those resulting from displacements and firings. Granted that some quits can be induced by the policy (in terms of salaries and benefits, for example) of firms that are weary of firing or laying off workers because of possible severance payments, but this calibration assumes that all quits are of this sort, completely ruling out quits for better paid jobs, which are quite common. Parrado and Wolff (1999) report that the wage earnings changes associated with all job separations that involve occupational changes are in fact positive.

The parameter values chosen to index the turbulence in the two steady-states, $\sigma(60s)$ and $\sigma(90s)$, imply that the average wage losses are 7 percent in the low turbulence steady-state and 10 percent in the high turbulence steady-state. This corresponds to a 43 percent growth in wage earnings losses, which is highly implausible.

The probability of a worker's skill level to advance is chosen such that a worker takes 7.3 years to reach the maximum skill level. As one can see from figure 3, this is a gross overestimation of the learning speed. The numbers implied by the CPS data are 23 years for college workers and 28 years for non-college workers.[38] I argue that this parameter alone drives the whole increase in cross-sectional dispersion. Such a high learning rate leads to a huge fraction of the population being bunched at the highest skill level. As a result, the initial distribution of wage earnings is extremely skewed. An increase in turbulence then

[38]These numbers are also supported by the findings of Murphy and Welch (1990). They find that maximum wage earnings are attained at 29 for college educated workers and 33 for non-college educated ones.

drives these individuals down the skill range, increasing dispersion.[39] If the initial earnings distribution were to be more disperse, decreasing the skill level would not necessarily result in more dispersion as it is clear from the previous experiments.

4.2 Increases in Turbulence and Occupational Mobility

In the previous section I assumed that the amount of occupational mobility did not change from one steady-state to another.[40] This was done with the purpose of focusing on the increase in turbulence as the sole driving force. In this section this assumption is relaxed and both factors are considered together.

There is a growing literature documenting the increase in occupational mobility from the late 1960s to the 1990s. Parrado and Wolff (1999) look at the PSID from 1969 to 1992 and conclude that, on average, workers shifted occupations 1.8 times in the 1969-80 period, and 2.1 times in 1981-92. This is an increase of 17 percent. Kambourov and Manovskii (2001) also use the PSID for the same period and conclude that the fraction of workers switching occupations each year increased from 10.2 percent in 1969 to 15.5 percent in 1993 at the one-digit level, an increase of 52 percent, and from 16 percent to 18.5 percent at the three-digit level, an increase of 16 percent.

The results these authors report are for all workers, meaning they include both workers that quit their job and changed occupations, as well as those that lost their job involuntarily, and then changed occupation (the ones of interest here). There are no results for this subgroup in particular, so I will assume that the increase in the fraction of job losers that switch occupations was the same as the increase in overall switching. Namely, I will assume that the increase in the fraction of switchers from one steady-state to the other was 17 percent, both for the fast learners and for the slow learners. This implies that the fraction of college educated switchers was 54.5 percent in the late 1960s, while this number was 57.9 percent for non-college educated workers. I then recalibrate the parameters of the model economy to match the same statistics as before. The parameters that change relative to the previous section are reported in table 10. Note that to keep the wage loss upon displacement constant

[39] There is another factor affecting wage earnings dispersion, which is the variation in the reservation wage schedule when turbulence increases, but this variation is negligible here.

[40] Occupational mobility is measured by the number of workers that change occupations as a fraction of total changes. Where occupations are defined by the 1970 Census of Population and provided by the PSID.

between the two steady-states, for each type, it was necessary to adjust the parameter indexing turbulence.

The results for this experiment are reported in table 11.[41] In terms of the college differential and cross sectional dispersion, including the change in occupational mobility does not add anything to the results from the previous section. In terms of the panel residual dispersion, it seems that including the change in occupational mobility is important. Figure 4 illustrates the changes in the dispersion of the transitory and permanent components of wage earnings for slow and fast learners. The black bars correspond to the steady-state with low turbulence and low occupational mobility, while the white bars correspond to the the steady-state with high turbulence and high occupational mobility. In particular, the results regarding the change in the dispersion of the permanent component of earnings are very encouraging. The changes considered account for half of the change in the data for the college educated workers and for one third for the non-college educated ones. The magnitudes for the change in the dispersion of the transitory component of earnings are more modest.

5 Conclusion

This paper examines whether an increase in economic turbulence as defined in Ljungqvist and Sargent (1998) can help us understand the observed increase in wage earnings dispersion both between and within educational groups in the U.S. economy.

Three fundamental points come out of this research. The first one is that indeed, increases in how fast tasks become obsolete are important in understanding increases in inequality between educational groups. As "the way things are done" changes more frequently, faster learners, because they are able to master new tasks more rapidly, experience an increase in their wage earnings premium over relatively slower learners. The second one is that in the context of a search model where workers accumulate skills, increases in turbulence do not seem to be important in understanding increases in cross-sectional inequality within educational groups. Third, and in contrast to the second point, this framework is helpful in accounting for changes in the dispersion of the permanent component of earnings, and to a lesser extent, in the dispersion of the temporary component of earnings.

[41] The change in turbulence considered was the same as in the first experiment of section 4.1, 10 percent.

The first result should be interpreted as complementary to the existing literature on the importance of skill-biased technological change. This result emphasizes how a particular kind of technological change (one that involves increases in the rate of technology obsolescence) changes the incentives to work, and thus accumulate human capital, for different educational groups, resulting in an increase in the wage earnings differential between them. This result does not rely on the fact that different educational groups supply different kinds of labor, rather, it relies on differences in learning speed between workers and how these differences interact with changes in the environment.

An important aspect of the increase in the college premium is that it was accompanied by an increase in the relative supply of college educated workers. In the model presented here, the decision to acquire more education is not present. A possible strategy to endogeneize this margin would be to have individuals choose (based on some exogenously drawn cost parameter) whether to acquire education or to work early in their lives and then have the skill accumulation speed depend on this choice.

The second and third results together emphasize the need for distinguishing between life-cycle effects and the effects resulting from an increase in turbulence. Quantitatively accounting for these two effects is an interesting question for future work.

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Table 1: **Selected White Male Earnings Statistics**

Variable	1963-69	1987-97	% Change
College Differential	0.388	0.483	**24.5**
sd(log(we_s))	0.412	0.504	**22.3**
sd(log(we_f))	0.480	0.522	**8.8**
Variable	1970-78	1979-87	% Change
var$(\mu_i)_s$	0.175	0.272	**55.4**
var$(\mu_i)_f$	0.184	0.200	**8.6**
var$(\nu_{it})_s$	0.106	0.208	**96.2**
var$(\nu_{it})_f$	0.065	0.093	**43.1**

Source: Author's calculations from the CPS and Gottschalk and Moffit (1994).

Table 2: **Three Year Displacement Rates**

	Switchers		*Non-switchers*	
Years	Non-coll.	Coll.	Non-coll.	Coll.
1981-83	0.093	0.041	0.040	0.021
1983-85	0.078	0.035	0.033	0.021
1985-87	0.068	0.032	0.033	0.023
1987-89	0.055	0.031	0.032	0.021
1989-91	0.083	0.051	0.044	0.027
1991-93	0.059	0.039	0.028	0.017
1993-95	0.054	0.038	0.029	0.020
1995-97	0.043	0.029	0.020	0.018
Average	0.067	0.037	0.032	0.021

Source: Author's calculations from the DWS.

Table 3: **Post-displacement Earnings Change**

	Switchers		*Non-switchers*	
Years	Non-coll.	Coll.	Non-coll.	Coll.
1981-83	-0.154	-0.028	-0.083	0.007
1983-85	-0.105	-0.057	-0.039	0.061
1985-87	-0.080	-0.099	-0.026	0.025
1987-89	-0.085	0.003	-0.007	-0.006
1989-91	-0.147	-0.102	-0.095	-0.107
1991-93	-0.168	-0.186	-0.094	-0.110
1993-95	-0.093	-0.086	0.023	-0.092
1995-97	-0.117	-0.011	-0.040	-0.020
Average	-0.119	-0.071	-0.045	-0.030

Source: Author's calculations from the DWS.

Table 4: **From Low Turbulence to High Turbulence: Calibration**

Parameter	Value	Fact Matched
$\alpha_s = \alpha_f$	0.00047	Average worklife 41 years
β	0.99925	Annual interest rate 4.0%
λ_s	0.00158	Three-year job loss rate 19.8 %
λ_f	0.00084	Three-year job loss rate 11.1%
$\sigma_s^{ts}(60s)$	0.20372	Wage loss of 10.8%
$\sigma_f^{ts}(60s)$	0.12884	Wage loss of 6.5%
$\sigma_s^{tns}(60s)$	0.08246	Wage loss of 4.1%
$\sigma_f^{tns}(60s)$	0.06083	Wage loss of 2.7%
$\sigma_s^{ts}(90s)$	0.23022	Wage loss of 11.9%
$\sigma_f^{ts}(90s)$	0.14318	Wage loss of 7.1%
$\sigma_s^{tns}(90s)$	0.08944	Wage loss of 4.5%
$\sigma_f^{tns}(90s)$	0.06481	Wage loss of 3.0%
γ_s	0.67700	Fraction of switchers 67.7%
γ_f	0.63800	Fraction of switchers 63.8%
$\pi_s^e(h,h)$	0.98645	28 years to reach h_{max}
$\pi_f^e(h,h)$	0.98355	23 years to reach h_{max}
$\pi_s^u(h,h)$	0.90000	Yearly skill loss between 0 and 18.3%
$\pi_f^u(h,h)$	0.90000	Yearly skill loss between 0 and 18.3%
$p_s(h_{min})$	0.08000	Log earnings of least skilled 5.46
$p_f(h_{min})$	0.40000	Log earnings of least skilled 5.81
μ_s^w	70.0000	Average log earnings 5.94
μ_f^w	118.000	Average log earnings 6.33
σ_s^w	100.000	No fact (Ljungqvist and Sargent (1998))
σ_f^w	100.000	No fact (Ljungqvist and Sargent (1998))

Table 5: **From Low Turbulence to High Turbulence: Results**

Variable	*Model* No Turbulence	Turbulence	% Change	*U.S. Economy* % Change
College Differential	0.390	0.400	**2.6**	**24.5**
sd(log(we_s))	0.363	0.362	**-0.3**	**22.3**
sd(log(we_f))	0.267	0.268	**0.4**	**8.9**
var$(\mu_i)_s$	0.089	0.095	**6.7**	**55.4**
var$(\mu_i)_f$	0.023	0.024	**4.3**	**8.6**
var$(\nu_{it})_s$	0.041	0.042	**2.4**	**96.2**
var$(\nu_{it})_f$	0.025	0.027	**8.0**	**43.1**

Table 6: **From No Turbulence to High Turbulence: Calibration**

Parameter	Value	Fact Matched
$\sigma_s^{ts}(90s)$	0.28810	Wage loss of 11.9%
$\sigma_f^{ts}(90s)$	0.13964	Wage loss of 7.1%
$\sigma_s^{tns}(90s)$	0.10000	Wage loss of 4.5%
$\sigma_f^{tns}(90s)$	0.06519	Wage loss of 3.0%
$p_s(h_{min})$	0.21000	Log earnings of least skilled 5.46
$p_f(h_{min})$	0.40000	Log earnings of least skilled 5.81
μ_s^w	20.0000	Average log earnings 5.94
μ_f^w	110.000	Average log earnings 6.33

Table 7: **From No Turbulence to High Turbulence: Results**

	Model			*U.S. Economy*
Variable	No Turbulence	Turbulence	% Change	% Change
College Differential	0.390	0.468	**20.0**	**24.5**
sd(log(we_s))	0.324	0.303	**-6.5**	**22.3**
sd(log(we_f))	0.266	0.267	**0.4**	**8.9**
var$(\mu_i)_s$	0.041	0.085	**107**	**55.4**
var$(\mu_i)_f$	0.023	0.027	**17.4**	**8.6**
var$(\nu_{it})_s$	0.043	0.053	**23.3**	**96.2**
var$(\nu_{it})_f$	0.032	0.032	**0.0**	**43.1**

Table 8: **Ljungqvist and Sargent (1998): Parameter Values**

Parameter	Value	Fact Matched
α	0.00045	Average worklife 42.7 years
β	0.99925	Annual interest rate 4%
λ	0.00450	Tenure before job loss 4.3 years
$\sigma(60s)$	0.14142	??
$\sigma(90s)$	0.20000	??
$\pi^e(h,h)$	0.95000	7.3 years to reach h_{max}
$\pi^u(h,h)$	0.90000	Yearly skill loss unemployed between 0 and 18.3%
μ^w	500.000	??
σ^w	100.000	??

Table 9: **Ljungqvist and Sargent (1998): Results**

Variable	Low Turbulence	High Turbulence	%Change
sd(log(we))	0.200	0.21	**5.0**
var(μ_i)	0.006	0.009	**50.0**
var(ν_{it})	0.016	0.018	**12.5**

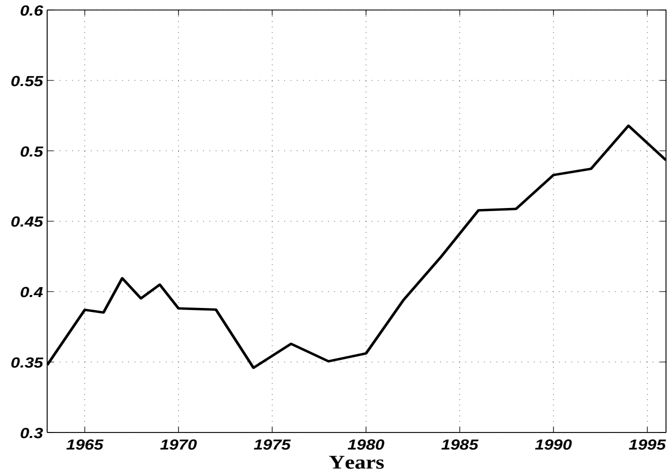
Table 10: **Increases in Turbulence and Occupation Mobility: Calibration**

Parameter	Value	Fact Matched
$\sigma_s^{ts}(60s)$	0.19235	Wage loss of 10.8%
$\sigma_f^{ts}(60s)$	0.12884	Wage loss of 6.5%
$\sigma_s^{tns}(60s)$	0.07937	Wage loss of 4.1%
$\sigma_f^{tns}(60s)$	0.06082	Wage loss of 2.7%
$\gamma_s(60s)$	0.57900	Fraction of switchers 57.9%
$\gamma_f(60s)$	0.54500	Fraction of switchers 54.5%

Table 11: **Increases in Turbulence and Occupation Mobility: Results**

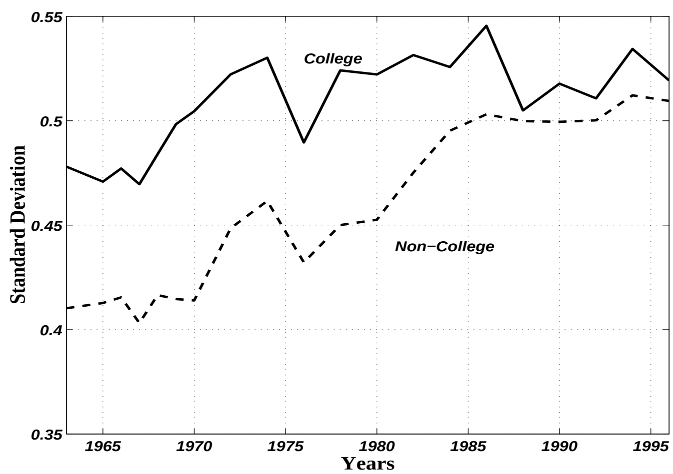
	Model			*U.S. Economy*
Variable	No Turbulence	Turbulence	% Change	% Change
College Differential	0.390	0.398	**2.1**	**24.5**
sd(log(we_s))	0.363	0.362	**-0.3**	**22.3**
sd(log(we_f))	0.267	0.268	**0.4**	**8.9**
var$(\mu_i)_s$	0.081	0.095	**17.3**	**55.4**
var$(\mu_i)_f$	0.023	0.024	**4.3**	**8.6**
var$(\nu_{it})_s$	0.040	0.042	**5.0**	**96.2**
var$(\nu_{it})_f$	0.026	0.027	**3.8**	**43.1**

Figure 1: **White Male College Differential**



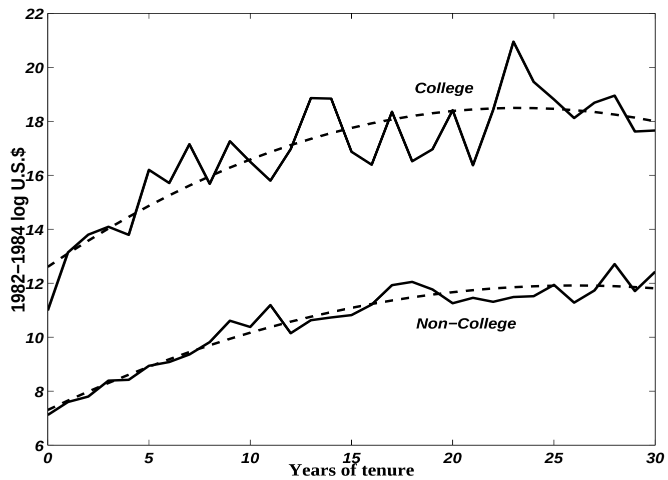
Source: See appendix.

Figure 2: **White Male Log Weekly Wage Earnings Residual Dispersion**



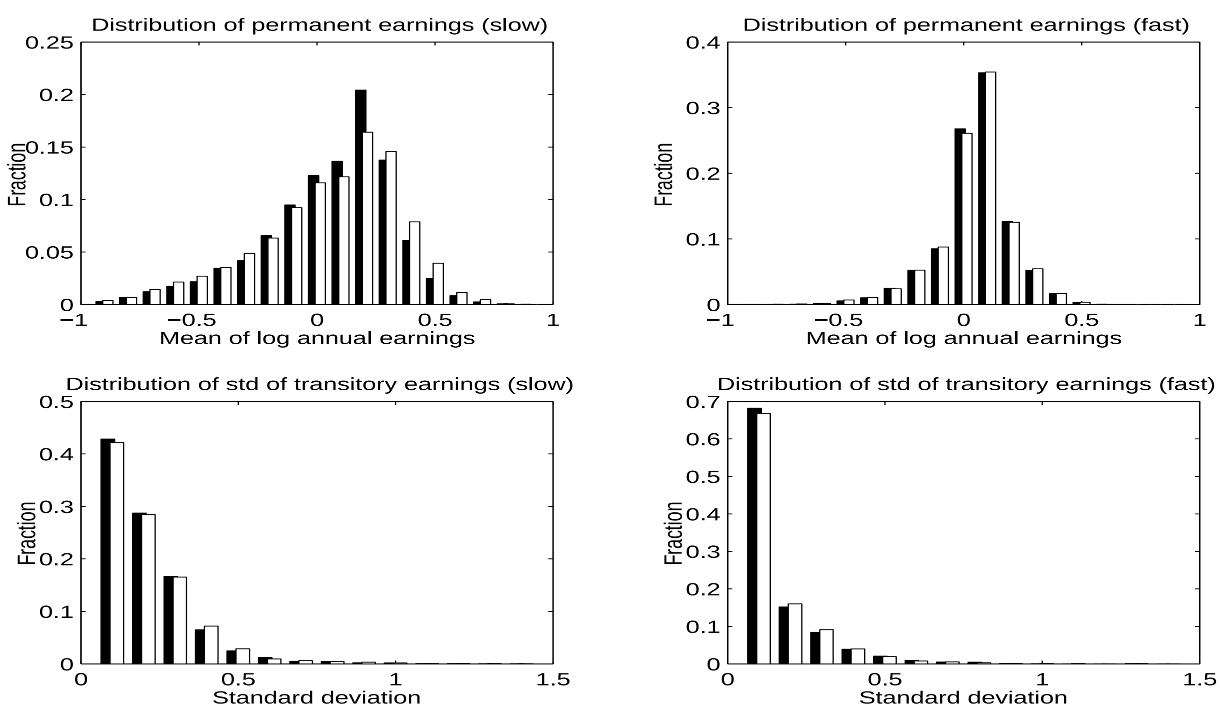
Source: See appendix.

Figure 3: **White Male Hourly Wage Earnings**



Source: See appendix.

Figure 4: **Increases in Turbulence and Occupation Mobility: Residual Earnings Dispersion**



6 Appendix

6.1 Data

Table 1: The college differential and the cross sectional standard deviations are computed averaging over the 1964, 1966, 1967, 1969, 1970, 1971, 1973, 1975, 1977, 1979, 1981, 1983, 1985, 1987, 1989, 1991, 1993, 1 and 1997 March CPS files. I consider full-time, full-year, white male workers between 20 and 64 years old, that are not self employed. A full time worker works at least 35 hours per week. A full year worker works at least 50 weeks a year. Following Katz and Murphy (1992), I exclude those workers earning less than half the 40 hour week equivalent of the 1982 minimum hourly wage, which was \$3.35. Top coded earnings were multiplied by 1.4. The variances of the permanent and transitory component of earnings are taken from table 1 in Gottschalk and Moffit (1994).

Tables 2 and 3: I use the 1984, 1986, 1988, 1990, and 1992 DWS supplements to the January CPS files and the 1994, 1996, and 1998 DWS supplements to the February CPS files. I consider the same sample as above.

There was a very important change in the survey. The surveys from 1984 to 1992 asked the workers if they were displaced from a job anytime in the preceding five-year period. The 1994, 1996, and 1998 surveys asked the workers if they were displaced from a job anytime in the preceding three-year period.[42] Since both surveys also asked in which year the displacement occurred, I will follow Farber (1997) and compute three-year displacement-switching, and displacement-non-switching, rates.[43]

Another problem has to do with the fact that the DWS reports information on at most one job loss for each worker. Some workers experience more than one job loss in a five-year or three-year period. For these workers the reported job loss refers to the longest job lost. In this sense, the DWS does not measure the total quantity of job losses, but the number of workers who have lost at least one job in the relevant period.

As all rates, the displacement rates I compute have a numerator and a denominator. For the denominator I would like to have the number of workers that are at risk of facing a displacement and posterior occupation switch (or not) during the three year time period. This is not easy to measure. I take this number to be the number of workers that are employed at the time of the survey. This approximation is a good one as long as the number of employed does not fluctuate a lot during the three-year period.

Computing the numerator is more complicated. The first problem that arises is that the only way to know if one worker is an occupation switcher or not is for her to be employed at the survey date. Naturally, some of the workers who get displaced are not employed at the survey date. To deal with this problem, I assume that the fraction of displaced workers unemployed at the time of the survey that are occupation switchers is the same as the fraction of displaced workers employed at the time of the survey that are occupation switchers. If anything, this simplifying assumption underestimates the true number of displaced switchers, since being unemployed might be precisely a consequence of the fact that the kind of task the worker specialized in has become obsolete.

[42] This change was made to address a severe recall bias problem identified in Topel (1990).

[43] Farber (1997) computes displacement rates that include both switchers and non-switchers. I will compute these rates separately for switchers and non-switchers

Another problem has to do with the adjustment necessary to convert from five-year rates to three-year rates in the 1984-92 surveys. As I mentioned, the survey asks the specific year the job loss took place, so it would seem that the solution would be to take into account only those workers who report job losses in the three years preceding the survey and count those that report losses four and five years before the survey as non job losers. There is, however, a serious comparability problem with this strategy. On one hand, the three-year job displacement rates computed from 1994 to 1998 include displacements reported in the three years before the survey that happened to workers that also got displaced four or five years before the survey. On the other hand, the three-year job displacement rates computed from 1984 to 1992 do not include these displacements if the displacement that occurred four or five years before the survey followed a longer tenure than the displacement that occurred in the three years before the survey.

I follow Farber (1997) in remedying this problem, by adjusting the three-year displacement rates from the 1984-92 surveys upward, to control for this downward bias. From the 1968-1985 Panel Study of Income Dynamics (PSID), Farber computes the fraction of workers who reported an involuntary job change four or five years before the survey date and subsequently reported an involuntary job change within the three years preceding the survey. Letting δ_4 denote the fraction of workers that lost a job four years before the survey and subsequently lost a job in the three periods before the survey, and δ_5 denote the fraction of workers that lost a job five years before the survey and subsequently lost a job in the three periods before the survey, he estimates that $\delta_4 = 0.3017$, and $\delta_4 = 0.2705$.[44]

The numbers found suggest that not considering this downward bias can seriously underestimate the amount of workers losing jobs in the three years before the survey, if one simply ignores those workers that report job losses four or five periods before the 1984-92 DWS surveys.

To summarize, let λ_i denote the three year displacement-switching rate in demographic group i. Let des_i^3 denote the sum of the number of workers that report a displacement in the first, second or third year before the survey, and also report an occupation change.[45] Let des_i^4, and des_i^5 denote the number of workers reporting a displacement in the fourth or fifth year before the survey, respectively, and also report an occupation change. Let du_i^3 denote the sum of the number of workers that report a displacement in the first, second or third year before the survey, and are unemployed at the date of the survey. Let du_i^4, and du_i^5 denote the number of workers reporting a displacement in the fourth or fifth year before the survey, respectively, and are unemployed at the date of the survey. Let e_i denote the number of employed workers at the time of the survey. Finally, let de_i^3, de_i^4, and de_i^5 denote the number of displaced and currently employed by year that the displacement was reported. Then, for the 1984-92 surveys, the rate of displacement-switching is:

$$\lambda_i = \left(\frac{des_i^3 + \delta_4 des_i^4 + \delta_5 des_i^5}{e_i} \right) + \left\{ 1 + \left(\frac{du_i^3 + \delta_4 du_i^4 + \delta_5 du_i^5}{de_i^3 + \delta_4 de_i^4 + \delta_5 de_i^5} \right) \right\} \tag{8}$$

[44] The structure of the PSID sample used is described in Farber (1997). Of particular interest is that the sample was 89.7 percent male. I will use the same numbers for the three demographic groups considered since Farber does not report any demographic breakdowns of these numbers.

[45] Therefore they are employed at the date of the survey,

For the 1994-98 surveys, that have only a three-year recall period, this rate is:

$$\lambda_i = \left(\frac{des_i^3}{e_i}\right) + \left\{1 + \left(\frac{du_i^3}{de_i^3}\right)\right\} \tag{9}$$

The rate of displacement-non-switching is computed in a similar fashion.

Figures 1 and 2: Same data as for table 1.

Figure 3: I use the 1987, 1991, and 1996, JTS supplements to the CPS. Same demographic sample as for table 1.

6.2 Proofs

The proofs are included for the referees' benefit and can easily be dropped from the paper since they are standard.

Proof. (proposition 6)

Pick any $(h, w) \in H \times \mathcal{W} \setminus \{0\}$. By assumptions 1 and 2, and because $0 < \alpha < 1$, and $0 < \beta < 1$, the two alternatives inside the max operator in the right-hand-side of equation 3 are bounded below by zero and above by $\frac{w_{max} h_{max}}{1-\beta(1-\alpha)}$. This guarantees existence.

Given that both alternatives inside the max operator are well defined, either one is different from the other, in which case the solution is unique, or they are equal. I will show that this happens at most in a set of measure zero. Let the second alternative inside the max operator be denoted $\overline{V}_i(h, w)$. Again, Pick any $(h, w) \in H \times \mathcal{W} \setminus \{0\}$ such that $V_i^u(h) = \overline{V}_i(h, w)$. Then, because $V_i^u(h)$ is constant in w, while $\overline{V}_i(h, w)$ is strictly increasing in w, for any $\epsilon > 0$, it follows that $\mid V_i^u(h) - \overline{V}_i(h, w + \epsilon) \mid > 0$. $\square$

Proof. (proposition 7)

Fix h. Guess that $V_i(h, \cdot)$ is nondecreasing in w.

The value the individual obtains when the job opportunity is rejected is constant in w. The value the individual obtains when the job opportunity is accepted is strictly increasing in w given the guess.

Let $\overline{V}_i(h, w)$ denote the value of accepting job opportunity w for an individual of type i and skill level h. It follows that if $V_i^u(h) \leq \overline{V}_i(h, w_{min})$, then, $\underline{w}_i(h) = w_{min}$, all offers are accepted and $V_i(h, w) = \overline{V}_i(h, w)$ for all $w \in \mathcal{W} \setminus \{0\}$. If, on the other hand, $V_i^u(h) > \overline{V}_i(h, w_{min})$, two things can happen. Either $V_i^u(h) \leq \overline{V}_i(h, w_{max})$, in which case $\underline{w}_i(h)$ is implicitly defined by the equation $V_i^u(h) = \overline{V}_i(h, \underline{w}_i(h))$, which has a unique solution given the assumptions, or $V_i^u(h) > \overline{V}_i(h, w_{max})$, in which case $\underline{w}_i(h) > w_{max}$, all offers are rejected, and $V_i(h, w) = V_i^u(h)$ for all $w \in \mathcal{W} \setminus \{0\}$.

The solution to (3) is then of the form:

$$V_i(h, w) = \begin{cases} \overline{V}_i(h, w) & w \geq \underline{w}_i(h) \\ V_i^u(h) & w < \underline{w}_i(h), \end{cases}$$

for $i = f, s$.

This solution is nondecreasing in w, confirming the guess. Noting that the solution to equation 3 is unique yields the result. $\square$

Proof. (proposition 8)

Fix h. For $w < \underline{w}_i(h)$, from proposition 7, $V_i(h,.) = V_i^u(h)$ is constant in w and $V_i(h, \underline{w}_i(h)) = V_i^u(h)$, so let $w \geq \underline{w}_i(h)$. For notational simplicity, drop the subscript i. Again, from proposition 7:

$$\begin{aligned} V(h,w) &= wh + \beta(1-\alpha)\Big[(1-\lambda)\sum_{h'} \pi^e(h,h')V(h',w) \\ &+ \lambda \sum_{h'} \pi^t(h,h')V^u(h')\Big]. \end{aligned} \tag{10}$$

The term in the second line is constant in w, so define:

$$c(h) = \beta(1-\alpha)\lambda \sum_{h'} \pi^t(h,h')V^u(h').$$

Using assumption 3, equation 10 can be rewritten as:

$$\begin{aligned} V(h,w) &= wh + \beta(1-\alpha)(1-\lambda)\Big[\pi^e(h,h)V(h,w) \\ &+ \big(1-\pi^e(h,h)\big)V(h_{+1},w)\Big] + c(h), \end{aligned}$$

where h_{+1} is the smallest element in H greater than h. This yields:

$$V(h,w) = \frac{wh + c(h)}{1-\beta(1-\alpha)(1-\lambda)\pi^e(h,h)} + \frac{\beta(1-\alpha)(1-\lambda)(1-\pi^e(h,h))}{1-\beta(1-\alpha)(1-\lambda)\pi^e(h,h)}V(h_{+1},w) \tag{11}$$

Equation 11 defines a linear operator in the space of real valued functions. Linear transformations of continuous piecewise linear functions are themselves continuous piecewise linear. It will now be established that $V(h_{max},.)$ is continuous and piecewise linear.

Suppose that $h = h_{max}$, then from assumption 3, and equation 11, for $w \geq \underline{w}(h_{max})$:

$$V(h_{max},w) = \frac{c(h_{max})}{1-\beta(1-\alpha)(1-\lambda)} + \frac{h_{max}}{1-\beta(1-\alpha)(1-\lambda)}\, w, \tag{12}$$

which is linear in w. This establishes that $V(h_{max},.)$ is linear in w for $w \geq \underline{w}(h_{max})$. From proposition 7, for $w < \underline{w}(h_{max})$, $V(h_{max},.) = V^u(h_{max})$, which is constant in w. Finally, note that $V(h_{max}, \underline{w}(h_{max})) = V^u(\underline{w}(h_{max}))$. This establishes that $V(h_{max},.)$ is piecewise linear in $w \in \mathcal{W} \setminus \{0\}$, and has the form:

$$V_i(h_{max},w) = \begin{cases} \frac{c(h_{max})}{1-\beta(1-\alpha)(1-\lambda)} + \frac{h_{max}}{1-\beta(1-\alpha)(1-\lambda)}\, w & w \geq \underline{w}_i(h_{max}) \\ V_i^u(h_{max}) & w < \underline{w}_i(h_{max}) \end{cases}$$

By definition of $\underline{w}_i(h_{max})$ and convexity of $\mathcal{W} \setminus \{0\}$, $V(h_{max},.)$ is also continuous in w.

The result follows from the operator defined by equation 11 and the fact that linear transformations of continuous piecewise linear functions are themselves continuous piecewise linear. $\square$

Proof. (proposition 10)

Existence and uniqueness of the optimal policies, $\underline{w}_i(h)$, were already shown in proposition 6. Existence and uniqueness of the associated invariant distributions will be shown here. The strategy of the proof will be as follows. I will start by assuming that the set $\mathcal{W}$ is finite and show that the result goes through, since this is the case I consider in the computations. I will then show that the result generalizes to the case where $\mathcal{W}$ is as in assumption 2.

Suppose $\mathcal{W}$ is finite. Assume there is just one type, as the proof is analogous for both types, and ignore the subscript i.

Let $S = H \times \mathcal{W}$ and let $l = \dim(S)$. Let $\mu \in \triangle^l = \left\{\mu \in \mathbb{R}^l : \mu \geq 0, \sum_{i=1}^{l} \mu_i = 1\right\}$, represent a measure on $(S, \mathcal{S})$, where $\mathcal{S}$ consists of all subsets of S.

Equations 5 and 6 define a linear transition function that can be represented by an $l \times l$ Markov matrix $\Pi = [\pi_{ij}]$, where $\pi_{i,j}$ represents the probability of moving from state s_i to state s_j, where s_i, $s_j \in S$.

A vector μ^* is said to be an invariant distribution if $\mu^*\Pi = \mu^*$.

Let $\pi_{ij}^{(n)}$ represent the probability of going from s_i to s_j in n steps. Let $\varepsilon_j^{(n)} = \min_i \pi_{ij}^{(n)}$ for $j = 1, ..., l$, be the minimum probability of getting to s_j in n steps. Finally, let $\varepsilon^{(n)} = \sum_{j=1}^{l} \varepsilon_j^{(n)}$.

Theorem 11.4 of Stokey, Lucas, and Prescott (1989), provides a necessary and sufficient condition under which an invariant distribution μ^* not only exists and is unique, but is the unique limit of the sequence $\{\mu_0 \Pi^n\}$ for any initial distribution μ_0.[46] Furthermore it guarantees that S has a unique ergodic set with no cyclically moving subsets. This result follows if and only if for some $N \geq 1$, $\varepsilon^{(N)} > 0$.

It is enough to show that there exists a state s_j that is eventually reached with strictly positive probability from every state s_i. By assumptions 3, 4, and 5, this is true of the state characterized by h_{max} and w_{max}, for example.

I will now show the result holds when $\mathcal{W}$ is of a more general form, as in assumption 2.

Let $(S, \mathcal{S})$ be a measurable space where $S = H \times \mathcal{W}$ and $\mathcal{S}$ be a $\sigma - algebra$ of the subsets of S. Let $\Psi(S, \mathcal{S})$ be the set of probability measures on $(S, \mathcal{S})$. Let $\psi \in \Psi(S, \mathcal{S})$.

Equations 5 and 6 define a transition function $P : S \times \mathcal{S} \to [0, 1]$. The interpretation is that for any $a \in S$ and $A \in \mathcal{S}$, $P(a, A)$ is the probability that the state next period is in A, given that the state this period was a. Associated with P, there is an operator $T^* : \Psi(S, \mathcal{S}) \to \Psi(S, \mathcal{S})$ defined by: $(T^*\psi)(A) = \int Q(a, A)\psi(da)$, for all $A \in \mathcal{S}$.

A probability measure ψ^* is said to be an invariant probability measure if, and only if, it is the fixed point of the operator T^*, that is: $\psi^* = T^*\psi^*$.

Theorem 11.12 of Stokey, Lucas, and Prescott (1989), provides a sufficient condition under which an invariant probability measure ψ^* exists and is unique. The result follows if there exists $\epsilon > 0$ and an integer $N \geq 1$ such that for any $A \in \mathcal{S}$, either $P^N(s, A) \geq \epsilon$, for all $s \in S$, or $P^N(s, A^c) \geq \epsilon$, for all $s \in S$, where $P^N(s, A)$ is the probability of getting from state s to a state in set A in N steps.

To show that this is true for this particular case, consider again the point in S that has $h = h_{max}$ and $w = w_{max}$, call it s_{max}.

First we need to find N^*. Pick any $s \in S$. We need to find how many steps it takes to get from s to s_{max} with strictly positive probability. Let l_h denote the number of elements in H. Starting at s,

[46] This convergence is element by element.

there is a strictly positive probability that in $N^* = l_h + 1$ steps the state is s_{max}. This result hinges on assumptions 3, 4, and 5. For states with $w = w_{max}$ and $h \neq h_{max}$ this is true for $N = h_{max} - h$. For states with $w = 0$ this is true for $N = l_h$. For states with $w \neq w_{max}$, but $w < \underline{w}(h)$, this is true for $N = l_h$. Finally, for states with $w \neq w_{max}$, but $w \geq \underline{w}(h)$, this is true for $N = l_h + 1$. The maximum over all these numbers is $N^* = l_h + 1$.

Next, let $\epsilon^* = inf\left(P^{N^*}(s, \{s_{max}\})\right)$. Clearly, by the reasoning above, $\epsilon^* > 0$.

Again, pick any $s \in S$ and any $A \in \mathcal{S}$, then, by construction, $P^{N^*}(s, \{s_{max}\}) \geq \epsilon^*$. Then, either $s_{max} \in A$ or $s_{max} \in A^c$, and the result follows. $\square$